Physical and Geometrical Aspects of Neutron Stars

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Abstract

The fundamental notions of neutron stars have been studied. Based on concepts and principles of astrophysics and cosmology, some physical and geometrical parameters have been calculated. Visualization of some results are done with Mathematica Software.

Keywords: neutron star, astrophysics, cosmology

Introduction

A neutron star, a very small and dense object which has a total mass of between 10 and 29 solar masses. Most of neutron stars are made up almost of entirely neutrons and a mass of 1.4 to 2.16 solar masses and the radius of these are typically 10 kilometers (km). The neutron stars are formed between the great explosion of some massive stars and the gravitational forces of other particles.

In the formation of them, these normally contain equal numbers of electrons and protons to be able to produce neutrons. To be a more massive neutron star, the collapse of neutron degeneracy pressure is mainly a part of it according to Pauli exclusion principle. If the needs are not fully provided, it becomes to form a black hole.

Regarding the neutron stars' temperature and pressure, they are incredibly hot and dense. Because of the great density, the star's weight is closely 3 billion tonnes. An ordinary neutron star's gravitational pull is examined as nearly 200 billion times stronger than earth's gravity. So, it can be said that to form a full neutron star, high temperature, density and pressure are needed [Potekhin A .Y, 2010].

Although some isolated neutron stars are not able to produce enough x- rays but others can do it with the help of having gravitational potential energy from the companion star that has enormously emitted electromagnetic radiation.

Figure 1 The picture of a neutron star – the star's tiny size and extreme density give it incredibly powerful gravity as its surface

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History of Discoveries

Sir James Chadwick, the first discoverer and the noble prize winner, found out the elementary particles called the neutrons in 1932. One year later, Walter Baade and Fritz Zwicky introduced the existence of neutron stars and particularly pointed out the powers of these stars. In 1967, Franco Pacini also revealed how these stars' electromagnetic waves were emitted and flowed inside their bodies. Then, Jocelyn Bell and Antony Hewish noticed the significant energy sources of the neutron stars which were also sending out regular radio pulses[Yakovlev D.G et al, 2004].

Formation of Neutron Stars

Before changing into a very energetic neutron star; firstly, the stars have run out of nuclear fuel in the core and it later must be provided by degeneracy pressure alone. Then, when these pressure were blown away, the core collapsed further and sent the combination of high temperature, density, electrons and protons as a supernova process. These processes are completely needed to form a fully neutron star[Gusakov M.E et al, 2004].

One more special point of a neutron star is having a strong, intense gravitational pull and magnetic fields which are much stronger than the earth's. Under these stars' influence of extraordinary gravity, most of the objects' elements are consumed.

Neutron Degeneracy

It is believed that Neutron degeneracy, as well as electron degeneracy, is greatly needed in the process of neutron stars. For weak points, any massive stars which have fewer masses can't be neutron stars as they have insufficient gravitational collapse[Yakovlev D.G et al, 2001].

On the contrary, the stars having enough masses can be neutron stars for having sufficient masses, pressure and gravitational collapse.

Gravitational Binding Energy for Uniform Sphere

Gravitational binding energy of a sphere with radius R is found by assuming a constant density ρ, the masses of a shell and the sphere inside it are:

$$m_{shell} = 4\pi r^2 \rho dr \qquad \text{and} \qquad m_{interior} = \frac{4}{3}\pi r^3 \rho$$

The required energy for a shell is the negative of the gravitational potential energy:

$$dU_g = -G\frac{m_{shell} \cdot m_{interior}}{r}$$

Integrating over all shells yields $$U_g = -G\int_0^R \frac{\left(4\pi r^2 \rho\right)\cdot\left(\frac{4}{3}\pi r^3 \rho\right)}{r}dr$$

$$= -G\frac{16}{3}\pi^2\rho^2\int_0^R r^4 dr = -G\frac{16}{3}\pi^2\rho^2\frac{R^5}{5}$$

$$\therefore U_g = -G\frac{16}{15}\pi^2\rho^2 R^5 \tag{1}$$

Since ρ is simply equal to the mass of the whole divided by its volume for objects with uniform density, therefore $\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3}$

(2) Substitute Equation (2) in Equation (1), $U_g = -G\frac{16}{15}\pi^2 R^5\left(\frac{M}{\frac{4}{3}\pi R^3}\right)^2$

$$\therefore U_g = -G\frac{3}{5}\frac{M^2}{R} = -\frac{3GM^2}{5R} \tag{3}$$

Then the variation of gravitational potential of neutron star is shown in Figure 1 and 2. These figures show that gravitational potential is gradually stable after passing the point of singularity.

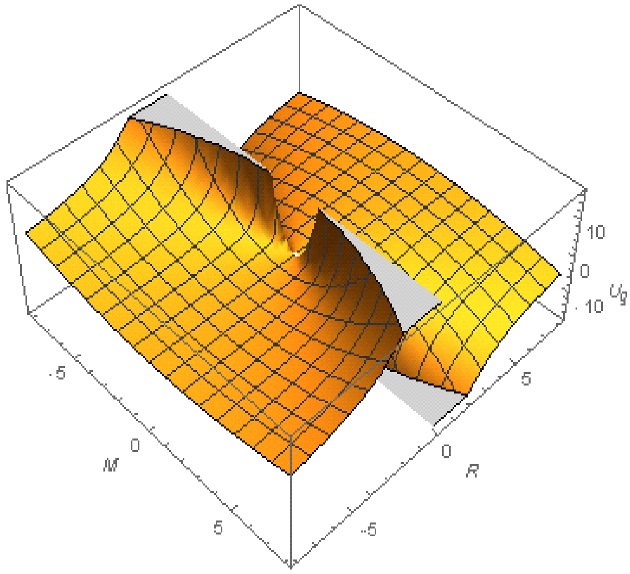

Figure 2 3D variation of gravitational potential with mass and radius of neutron star

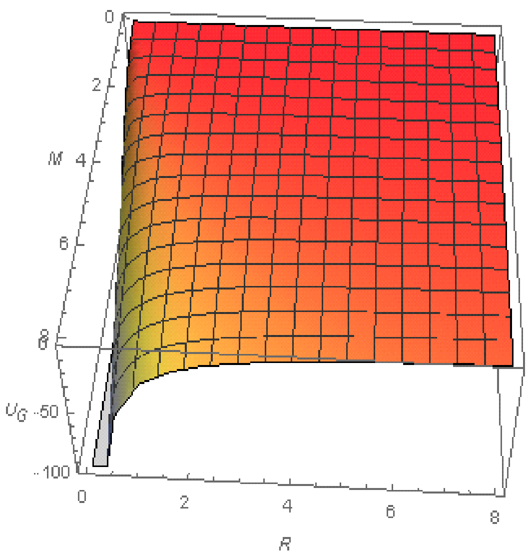

Figure 3 3D variation of gravitational potential with mass and radius of neutron star avoid singularity point

Calculation of Degeneracy Pressure and Mass-Radius Relation

(i) Fermi Energy

Starting by calculating the quantum number magnitude $n_F = \sqrt{n_x^2 + n_y^2 + n_z^2}$ of the neutron at the very topmost fufilled energy level. This assumes zero temperature of the neutrons, which is not entirely accurate, but is a reasonable approximation. The formula relating the total number of neutrons N to n_F is:

$$N = 2 \cdot \left(\frac{1}{8}\right)\left(\frac{4}{3}\pi n_F^3\right) \tag{4}$$

The factor of 2 represents the fact that both spin up and spin down are available in each quantum state.

$$N = \frac{1}{3}\pi n_F^3$$

$$n_F = \left(\frac{3N}{\pi}\right)^{\frac{1}{3}} \tag{5}$$

At a given energy level, a neutron has a momentum $|p| = \dfrac{hn}{2V^{1/3}}$ corresponding to the nth wave solution in quantum mechanics. It can use to compute the Fermi Energy:

$$\varepsilon(n) = \frac{|p|^2}{2m} = \frac{h^2|n|^2}{8mV^{2/3}} \tag{6}$$

$$\varepsilon_F = \varepsilon(n_F) = \frac{h^2 n_F^2}{8mV^{2/3}} = \frac{h^2\left(\frac{3N}{\pi}\right)^{2/3}}{8mV^{2/3}}$$

$$\therefore \varepsilon_F = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3} \tag{7}$$

The symbol m denotes the neutron mass, not the electron mass. Then the variation of Fermi Energy with volume and total number of neutrons are demonstrated in Figure 4. This figure shows that the Fermi Energy is decreased due to the increasing of volume of neutron star.

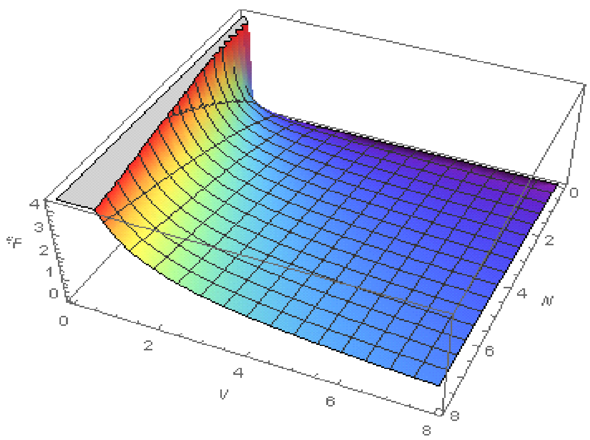

Figure 4 Demonstration of Fermi energy with volume of neutron star and total number of neutrons

(ii) The internal Energy of the Core

Now computing the internal energy of the core, excluding gravitational energy. This is done by integrating the energies of all of the fulfilled energy levels. Since the core is degenerate, these levels begin at the ground state and are continuously fulfilled up to n_F .

$$U = 2\int\int\int \varepsilon(n) d^3 n \text{ (both spin up and down)}$$

$$= 2\int_0^{n_F}\int_0^{\pi/2}\int_0^{\pi/2} \frac{h^2 n^2}{8mV^{2/3}} n^2 \sin\theta \cdot dn d\theta d\phi = 2\int_0^{n_F}\int_0^{\pi/2}\int_0^{\pi/2} \frac{h^2 n^4}{8mV^{2/3}} \sin\theta \cdot dn d\theta d\phi$$

$$= 2\int_0^{n_F}\int_0^{\pi/2} \frac{h^2 n^4}{8mV^{2/3}} \sin\theta\left(\frac{\pi}{2}\right) \cdot dn d\theta = 2\left(\frac{\pi}{2}\right)\frac{h^2}{8mV^{2/3}} \cdot \int_0^{n_F} n^4 \left[-\cos\theta\right]_0^{\pi/2} dn$$

$$= \frac{h^2\pi}{8mV^{2/3}} \int_0^{n_F} n^4 (0-(-1)) dn = \frac{h^2\pi}{8mV^{2/3}} \int_0^{n_F} n^4 dn$$

$$= \frac{h^2\pi}{8mV^{2/3}} \cdot \frac{n_F^5}{5} = \left(\frac{h^2 n_F^2}{8mV^{2/3}}\right) \cdot \left(\frac{\pi n_F^3}{5}\right) = \left(\frac{h^2 n_F^2}{8mV^{2/3}}\right) \cdot \left(\frac{\pi n_F^3}{3}\right) \cdot \left(\frac{3}{5}\right)$$

$$\therefore U = \frac{3}{5} N\varepsilon_F \tag{8}$$

(iii) Degeneracy Pressure

From the above expression for the internal energy, it can compute the degeneracy pressure in the usual thermodynamic way: $P_d = -\frac{\partial U}{\partial V}$

$$P_d = -\frac{\partial}{\partial V}\left[\frac{h^2\pi}{8mV^{2/3}} \cdot \frac{n_F^5}{5}\right] = -\left[-\frac{2}{3} \cdot \frac{h^2\pi}{8mV^{5/3}} \cdot \frac{n_F^5}{5}\right]$$

$$= \frac{2}{3} \cdot \frac{h^2\pi}{8mV^{5/3}} \cdot \frac{n_F^5}{5} = \frac{2}{3} \cdot \frac{h^2\pi}{8mV^{5/3}} \cdot \frac{1}{5}\left(\frac{3N}{\pi}\right)^{5/3}$$

$$\therefore P_d = \frac{2}{5}\frac{h^2}{8m}\left(\frac{3}{\pi}\right)^{2/3} \cdot \left(\frac{N}{V}\right)^{5/3} \tag{9}$$

Next turning to the gravitational pull of the star upon itself, as derived in the equation (3), its gravitational self-energy is given, in the Newtonian Theory, by:

$$U_g = -\frac{3}{5}\frac{GM^2}{R}$$

$$= -\frac{3}{5} GM^2 \left(\frac{4\pi}{3V}\right)^{1/3} \quad \left(\because V = \frac{4}{3}\pi R^3\right) \tag{10}$$

(iv) The Gravitational Pressure

The gravitational pressure is calculated as before:

$$P_g = -\frac{\partial U_g}{\partial V} = -\frac{\partial}{\partial V}\left[-\frac{3}{5}GM^2\left(\frac{4\pi}{3V}\right)^{1/3}\right] = \frac{3}{5}GM^2\left(\frac{4\pi}{3}\right)^{1/3}\frac{\partial}{\partial V}\left(V^{-1/3}\right)$$

$$P_g = \frac{3}{5}GM^2\left(\frac{4\pi}{3}\right)^{1/3}\left(-\frac{1}{3}\right)V^{-4/3}$$

$$\therefore\ P_g = -\frac{GM^2}{5}\left(\frac{4\pi}{3}\right)^{1/3}V^{-4/3} \tag{11}$$

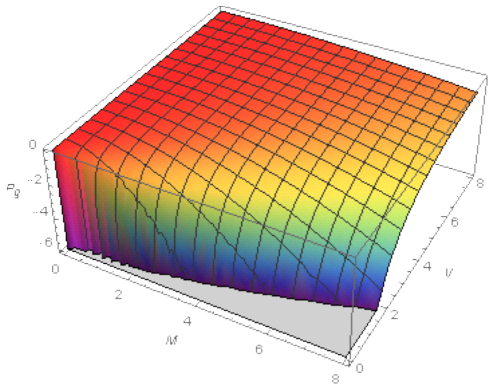

Figure 5 Gravitational pressure with mass and volume

The negative value indicates that the pressure is attempting to compress the star instead of to expand it. The pressures are balanced in the case of neutron star:

$$P_d + P_g = 0 \tag{12}$$

By making the substitution of the values of P_d and P_g from the equation (9) and the equation (11),

$$\frac{2}{5}\frac{h^2}{8m}\left(\frac{3}{\pi}\right)^{2/3}\left(\frac{N}{V}\right)^{5/3} - \frac{GM^2}{5}\left(\frac{4\pi}{3}\right)^{1/3}V^{-4/3} = 0 \tag{13}$$

where m is the mass of a neutron and the neutron star's total mass is simply the multiple of m and the total number of neutrons, N present in the neutron star, $M = mN$,

$$\frac{2}{5}\frac{h^2}{8m}\left(\frac{3}{\pi}\right)^{2/3}\left(\frac{N}{V}\right)^{5/3} = \frac{GM^2}{5}\left(\frac{4\pi}{3}\right)^{1/3}V^{-4/3}$$

$$\frac{2}{8}\frac{Nh^2}{M}\left(\frac{3}{\pi}\right)^{2/3}\left(\frac{N}{V}\right)^{5/3} = \frac{GM^2}{5}\left(\frac{4\pi}{3}\right)^{1/3}V^{-4/3}$$

$$\frac{2}{8}h^2\left(\frac{3}{\pi}\right)^{\frac{2}{3}}\left(\frac{3}{\pi}\right)^{1/3}\left(\frac{1}{V}\right)^{5/3}N^{8/3} = 4^{1/3}GM^3V^{-4/3}$$

$$\frac{2}{8}h^2\left(\frac{3}{\pi}\right)N^{8/3} = 2^{2/3}GM^3V^{1/3}$$

$$\frac{2}{8}\cdot\frac{1}{2^{2/3}}\times\frac{h^2}{GM^3}\left(\frac{3}{\pi}\right)N^{8/3} = V^{1/3}$$

$$\frac{1}{2^{8/3}}\frac{h^2}{GM^3}\left(\frac{3}{\pi}\right)N^{8/3} = V^{1/3}$$

$$\frac{1}{2^{8/3}}\frac{h^2}{GM^3}\left(\frac{3}{\pi}\right)\left(\frac{M}{m}\right)^{8/3} = V^{1/3}$$

$$\frac{1}{2^{8/3}}\frac{h^2}{Gm^{8/3}}\left(\frac{3}{\pi}\right)M^{-1/3} = V^{1/3} \tag{14}$$

(v) Theoretical value for the core's radius

To derive a theoretical value for the core's radius at equilibrium:

$$R = \left(\frac{3}{4}\frac{V}{\pi}\right)^{1/3} = \left(\frac{3}{4\pi}\right)^{1/3}V^{1/3} = \left(\frac{3}{4\pi}\right)^{1/3}\cdot\frac{1}{2^{8/3}}\frac{h^2}{Gm^{8/3}}\left(\frac{3}{\pi}\right)M^{-1/3}$$

$$R = \frac{3^{4/3}}{2^{10/3}}\frac{h^2}{Gm^{8/3}\pi^{4/3}}M^{-1/3} \tag{15}$$

where R = radius of the neutron star , M = mass of the neutron star

Pd = degeneracy pressure of the neutron star, m = mass of a neutron (1.6749×10^{-27} kg)

G = gravitational constant (6.674×10^{-11} $m^3kg^{-1}s^{-2}$)

h = Planck's constant ($6.62607015 \times 10^{-34}$ Js) Pg = gravitational pressure of the neutron star

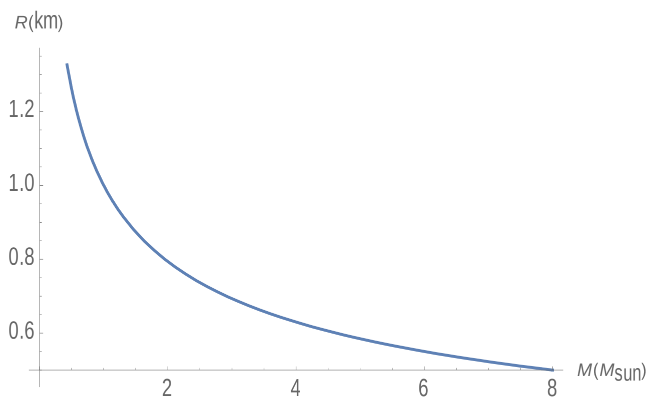

Figure 6 Graph of the radius R with mass M of the neutron star

Assuming that the core has a mass of M = 1.5, M_{Sun} = 3 × 10^{30} kg, it arrives at an estimate of a neutron star's radius and degeneracy pressure. From equation (15) and equation (9), R = 10.755 km and P_d = 2.16 × 10^{33} Pa. Then, the relation between the radius and mass are shown in Figure 6. From this, one can conclude that the larger the mass the smaller the radius.

Conclusion

It is concluded that a giant star of total mass between 10 and 29 solar masses has the potential to become a neutron star. A typical neutron star has a radius in the order of 10 kilometers and mass between 1.4 and 2.16 solar masses. As a basic model of the neutron star, it can assume that neutron star is composed mostly out of neutrons; the electrons and protons present in normal matter combine to produce neutrons at the conditions in a neutron star. Neutron stars that can be observed are very hot and have a surface temperature of around 600000 K. A neutron star of a normal-sized can weigh of approximately 3 billion tonnes. The gravitational field at the neutron star's surface is about 2×10^{11} (200 billion) times that of the Earth. The neutron stars have an escape velocity ranging from 100,000 km/s to 150,000 km/s, that is, from a third to half the speed of light. When a massive star is compressed during a supernova, and collapse into a neutron star, it retains most of its angular momentum. But, it has only a tiny fraction of its parents radius and therefore a neutron star is formed with very high rotation speed due to the conservation of angular momentum; in analogy to spinning ice skaters pulling in their arms. Neutron stars are known that can have rotation periods from about 1.4 ms to 30 ms. Then, it is calculated various properties of neutron stars. After that it is derived the gravitational binding energy. Finally, it is shown the degeneracy pressure and mass-radius relation of the neutron star. As the radius of the neutron star gets smaller, the compactness parameter and the density get bigger.

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